On the relationship between C_n^2 and humidity

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ABSTRACT

We have recently shown the refractive index structure constant C_n^2 in the visible and near infrared to be a strong function of humidity in the absence of solar insolation effects, in stark contrast to the commonly held assumption that the humidity contribution can be ignored in that waveband. We expand our analysis of the effects of humidity on C_n^2 as measured across a 100-m long horizontal beam path to include temperature. Also we present a new technique for extracting information on changes in the parameter space of C_n^2 and local weather variables, which we term Hilbert Phase Analysis (HPA). This methodology, based on extracting the phase of the analytic signal via Hilbert transforms, reveals a wealth of detail that conventional analysis techniques cannot determine. The HPA provides additional confirmation that C_n^2 is strongly influenced by local humidity in the visible region. We have also found that HPA provides a clear demonstration that humidity competes with temperature in affecting the value of C_n^2.

Keywords: Hilbert Phase Analysis, Humidity, Refractive Index Structure Function

1. INTRODUCTION

During 2003 and 2004, measurements were taken of path integrated C_n^2 and a host of local climate parameters using commercially available instruments, as discussed in Font[1] *et al.* We present follow–up analysis of the datasets presented in the aforementioned work in this paper, wherein we identify system changes through Hilbert Transform based phase analysis or HPA.

1.1. HILBERT ANALYSIS OF TIME SERIES

Gabor defined the complex analytic signal, namely

$$\begin{aligned} \Psi(t) &= X(t) + iY(t) \qquad (1)\\ \text{where } Y(t) &= \mathcal{H}[X(t)] \\ &= \frac{-1}{\pi}\mathbf{P}\int_{\Omega}\frac{X(t)}{(s-t)}ds\ , \quad t\in\Omega \end{aligned}$$

where $\mathcal{H}[\bullet]$ represents a Hilbert Transform. As a result, $\Psi(t)$ is unique and an analytic signal. The Hilbert Transform is a well known integral transform with a singular kernel $(1/(\pi(t-s))$, s also being a time variable if t is time. As a result it is also a Cauchy Principal integral, which we denote by $\mathbf{P}$, where the real axis singularity at $t = s$ is taken along a positive semi-circular path. An alternative way of writing Equation 1 is

$$\begin{aligned} \Psi(t) &= a(t)\exp^{i\Phi(t)}\ , \quad \text{whence} \qquad (2)\\ a(t) &= \sqrt{X^2(t)+Y^2(t)} \\ \Phi(t) &= \arctan\left(\frac{Y(t)}{X(t)}\right) \end{aligned}$$

This de Moivre form of the analytic signal is similar to the Fourier amplitude and phase expression. Note though that the Hilbert amplitude and phase are time dependent variables, as opposed to the Fourier analysis where they are fixed values per frequency.

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1.2. A PHYSICAL HILBERT TRANSFORMATION

If one applies the Hilbert Transform directly to a time varying signal, there is a high probability that at least one of five paradoxes[2] will be encountered, leading to unphysical results, due to the presence of so–called "riding waves" which cause asymmetries in the signal.

These paradoxes may be avoided by the application of Empirical Mode Decomposition[3] (EMD) method developed in a seminal work by Huang *et al.*[4] From the application of the EMD method, we extract Intrinsic Mode Functions (IMFs) whose instantaneous frequencies are well defined.

The application of the Hilbert Transform to the IMFs yields physically meaningful interpretations of the oscillatory phenomena. This may be best appreciated if we consider the Hilbert phase space, as in Figure 1.

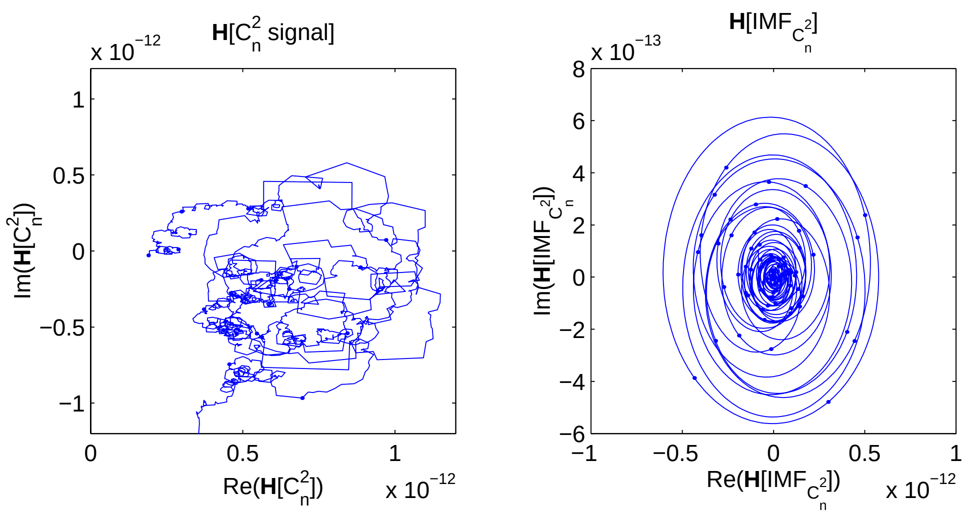

Figure 1. (a) Hilbert Phase Space plot of recorded C_n^2 signal, (b) Hilbert Phase Space plot of an example IMF of the C_n^2 signal.

By naïvely applying the Hilbert Transform to a non–stationary time series (C_n^2 measurements) we see that the trajectory of the analytic signal's vector is subject to many alterations in origin and phase angle. The instantaneous frequency, defined as

$$\omega = \frac{d\Phi}{dt} \tag{3}$$

can have both positive and negative values, rendering the Hilbert transform physically uninterpretable. The Hilbert Transform of the IMFs, of which an example is shown on the right side of Figure 1, ensures that the analytic signal vector's origin stays fixed and no sudden changes in the direction of ω occur.

1.3. HILBERT PHASE ANALYSIS

The HPA technique is based on the conditions mentioned in the previous subsections. The IMFs determined through EMD from the input signal are the analytic signal eigenfunctions of the EMD sifting operation and it is clear that a phase angle ($\Phi(t)$), as well as an amplitude ($a(t)$), can be determined from them. Physical effects of a non–linear, non–stationary time varying system can be studied by summing the $\Phi(t)$s of all the IMFs. It should be noted that the operations

$$\mathbf{H}\left[\sum \mathrm{IMF}\right] \quad \text{and} \quad \sum \mathbf{H}\left[\mathrm{IMF}\right] \tag{4}$$

are commutable, as may be seen in Figure 2. The term $\mathbf{H}[\bullet]$ represents the Hilbert Transform.

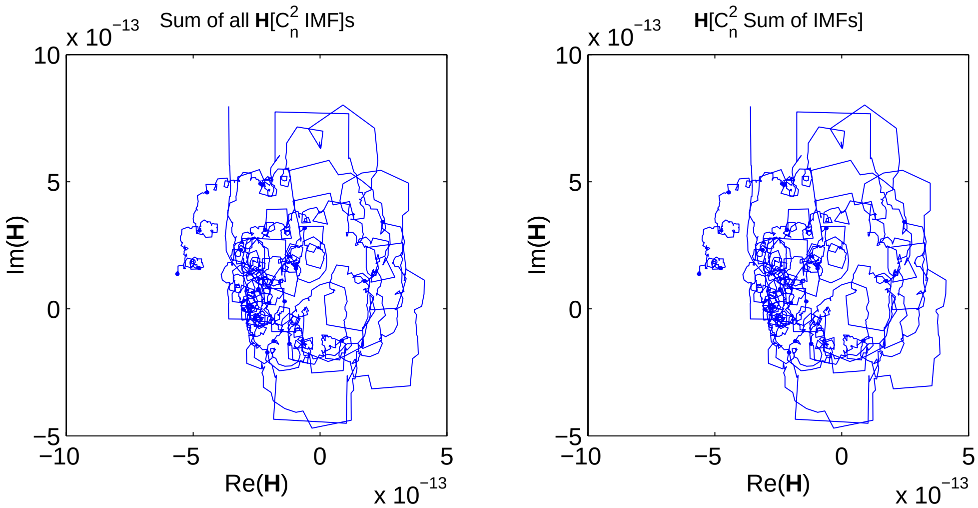

Figure 2. Plots illustrating the commutability of operations.

2. DATA

2.1. HPA PROTOCOL

The procedure for the HPA data analysis is as follows:

1. Firstly, we decompose the C_n^2, humidity and temperature time series measured at the Cheasapeake Bay Detachment (CBD) facility of the Naval Research Laboratory via EMD, obtaining their IMFs and trend lines.
2. Next we apply the Hilbert Transform to the various IMF sets, taking care to exclude the trend lines.
3. As a final step, we sum the phase angle $\Phi(t)$ of the Hilbert Transformed IMFs.

2.2. RESULTS

The Hilbert phase angles $\Phi(t)$ are plotted for each of the time series: C_n^2, humidity and temperature in Figures 3 to 10. We also include the difference between pairs of phase angle series: $\Phi_C - \Phi_H$, $\Phi_C - \Phi_T$ and $\Phi_C - \Phi_{(H+T)/2}$.

3. DISCUSSION

Upon inspecting the Hilbert phase of the C_n^2 IMFs, Φ_C, we see that the gradients of the various functions are not constant, although there is an increasing trend in all the plots. The Hilbert phase functions of humidity and temperature, Φ_H, Φ_T respectively, also show an increase (except for one case, where the temperature function produced only the trend) with time, although their gradients vary markedly between data sets.

The straightforward average function of the humidity and temperature phase angles are all approximately linear (and increasing) over time in all cases. This occurs despite the variations in the individual Φ_H and Φ_T; the conclusion that we may draw from this observation is that the humidity and temperature are approximately inversely related during the time intervals of these data sets.

To better understand the dependence of Φ_C upon humidity and temperature, we turn to the plots of the difference functions against time. The overall linearity of the Φ_C and $(\Phi_H + \Phi_T)/2$ functions might lead one to expect that $[\Phi_C - (\Phi_H + \Phi_T)/2]$ would be constant as a function of time. The actual graphs give the lie to that expectation. Instead what is seen is the influence of different factors competing over time, a fact that is deduceable by comparison with the $\Phi_C - \Phi_H$ and $\Phi_C - \Phi_T$ graphs.

As a result of inspecting the various difference functions, we conclude the following,

- November 3, 2003 (night) : temperature dominates the C_n^2 behaviour.
- November 9, 2003 (morning) : both temperature and humidity vie for dominance over the time interval of this sample.
- November 10, 2003 (morning) : temperature dominates.
- February 2, 2004 (morning) : the earlier half is dominated by temperature and the latter half is dominated by humidity.
- March 27, 2004 (night) : temperature dominates.
- March 28, 2004 (morning) : humidity dominates.
- April 3, 2004 (morning) : temperature dominates.
- April 3, 2004 (night) : humidity dominates.

4. CONCLUSIONS

We have described the Hilbert Phase Analysis, a new technique for determining the characteristics of multiple, non–stationary time series data and their interrelated effects. We have found that the contribution of humidity to C_n^2 is significant and cannot be disregarded, confirming our earlier work. We have also determined that temperature had an approximate inverse relationship to humidity in the measurements taken. Finally, and most significantly, we have shown that the HPA methodology is able to discriminate between the competing influences of humidity and temperature on the behaviour of C_n^2.

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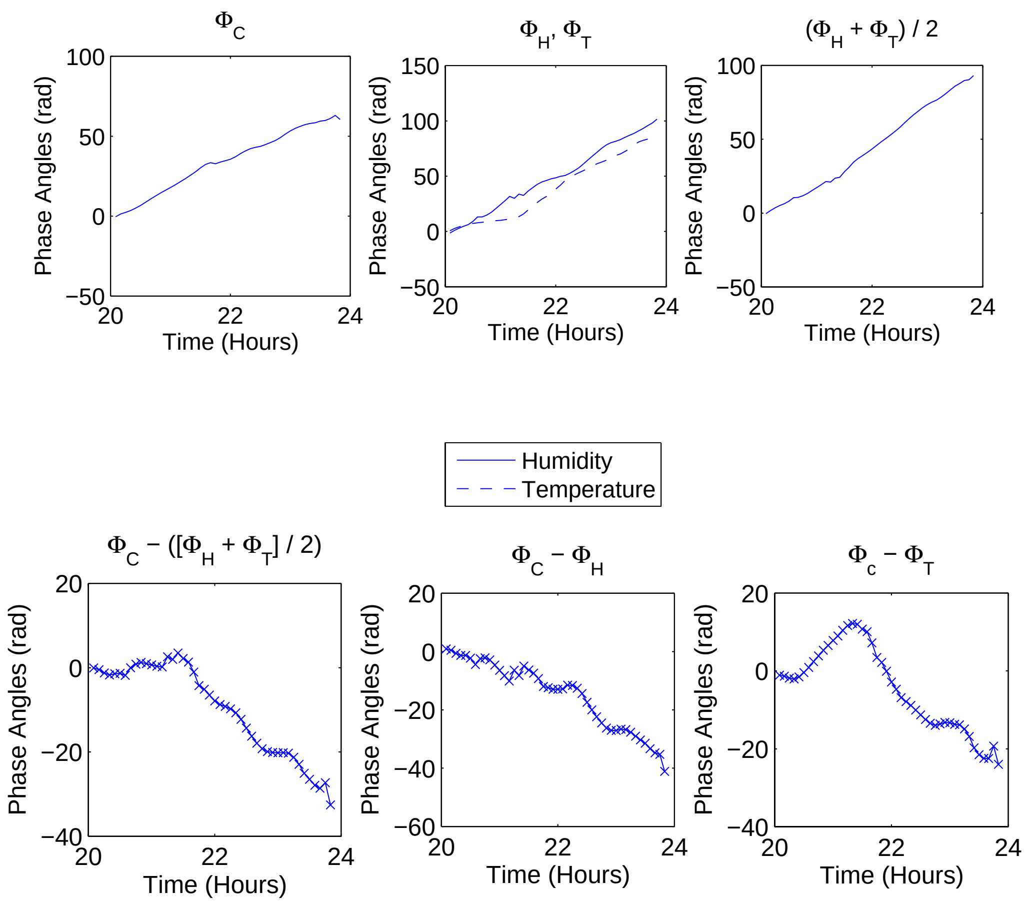

Figure 3. November 3, 2003 night. Temperature dominates.

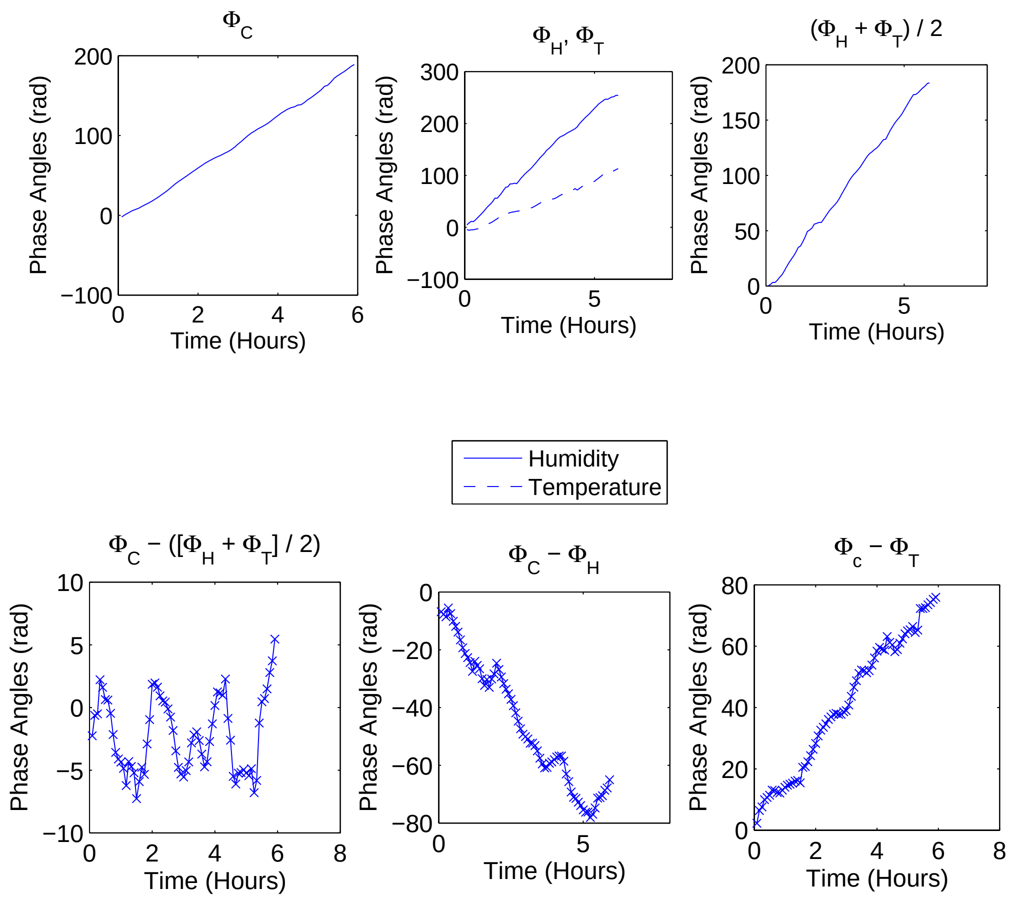

Figure 4. November 9, 2003 morning. Temperature and humidity are seen to compete for influence on the C_n^2 function.

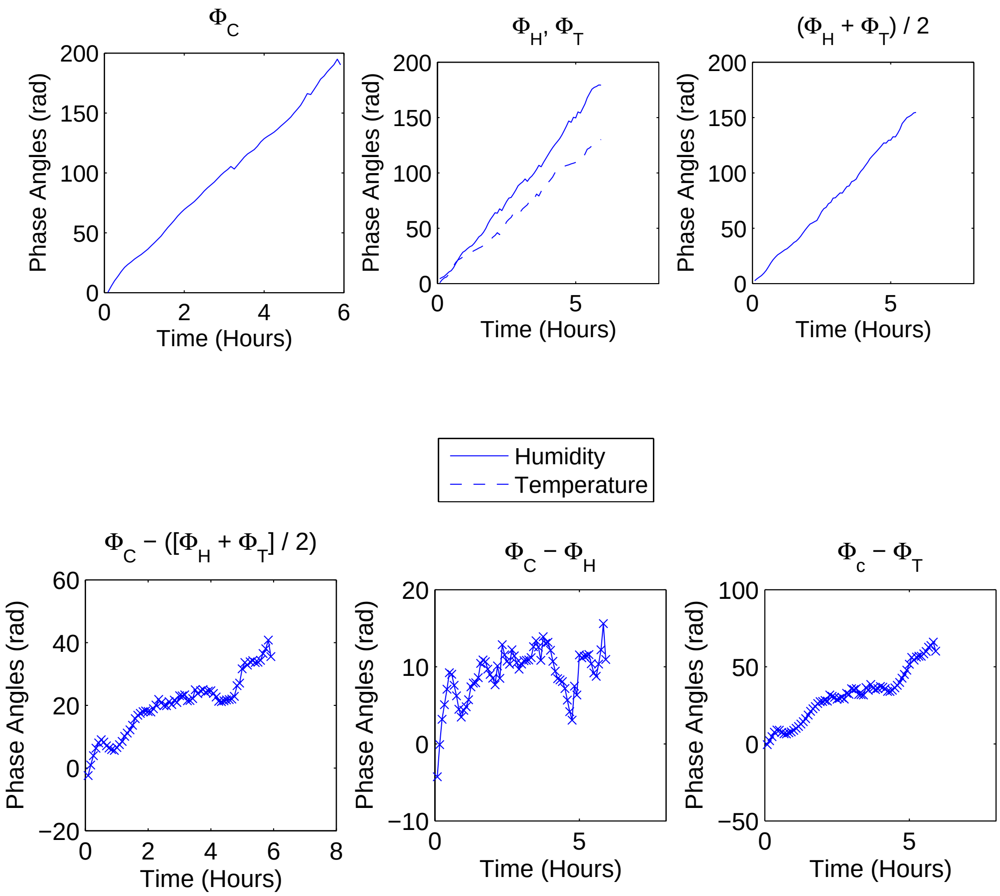

Figure 5. November 10, 2003 morning. Temperature dominates.

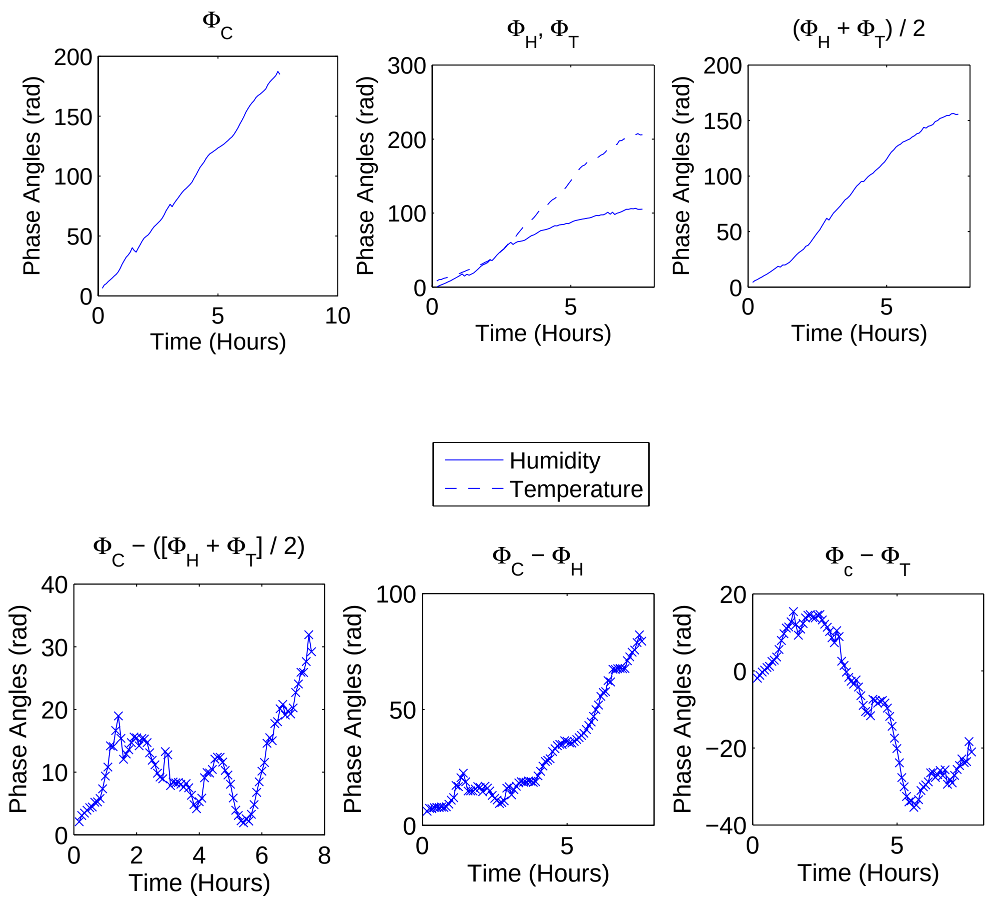

Figure 6. February 2, 2004 morning. Temperature dominates in the early stage, then humidity takes over.

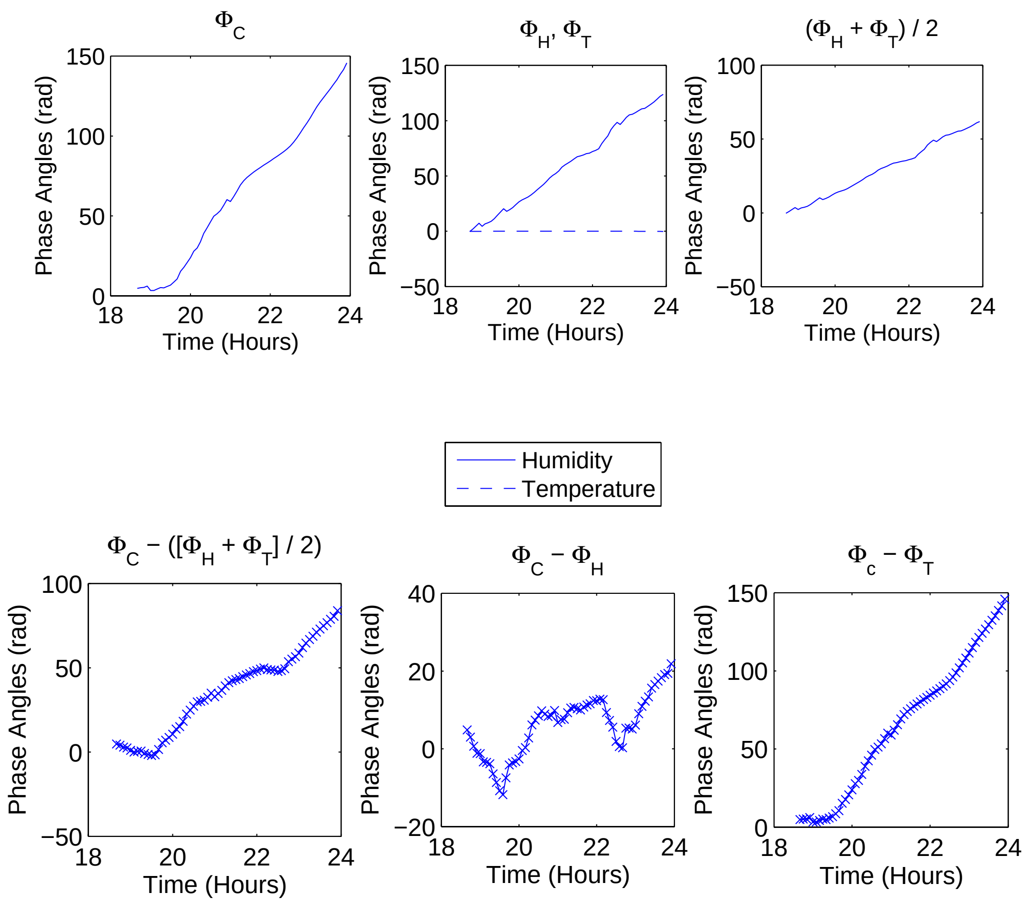

Figure 7. March 27, 2004 night. The temperature phase plot shows a constant phase angle since the original data was in itself an IMF, therefore EMD extracted only a single IMF. Naturally temperature dominates.

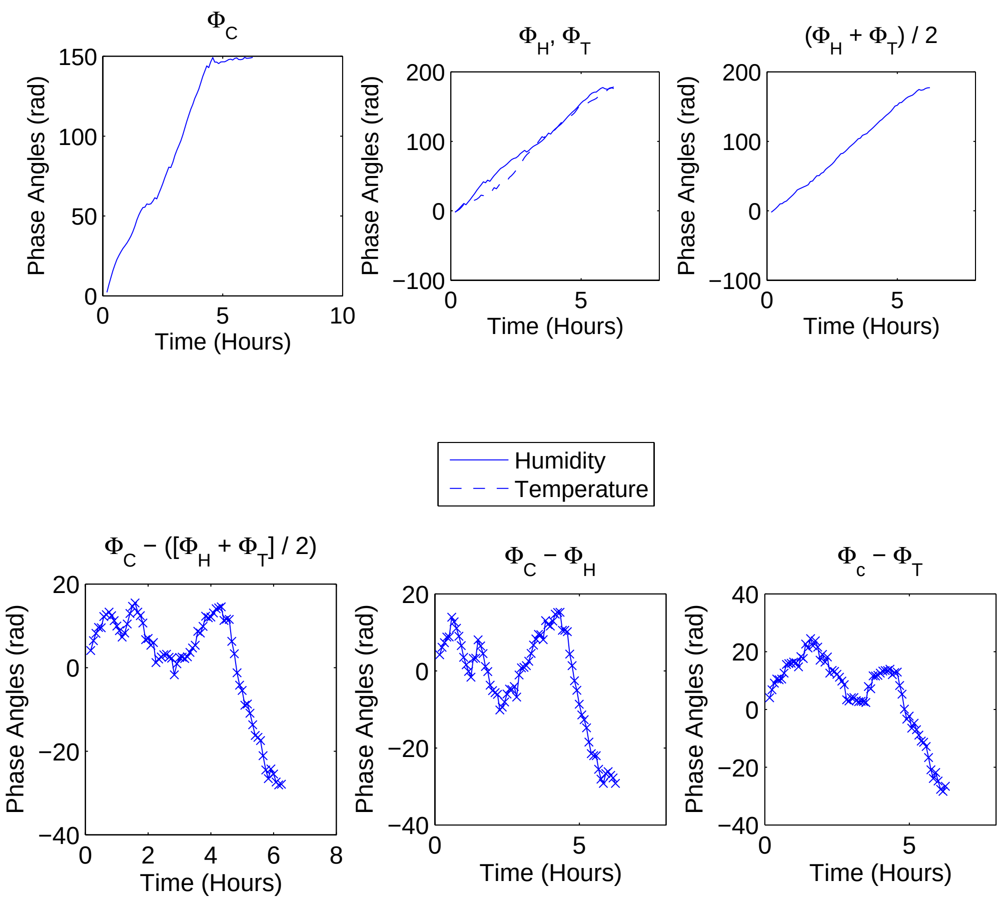

Figure 8. March 28, 2004 morning. Humidity dominates.

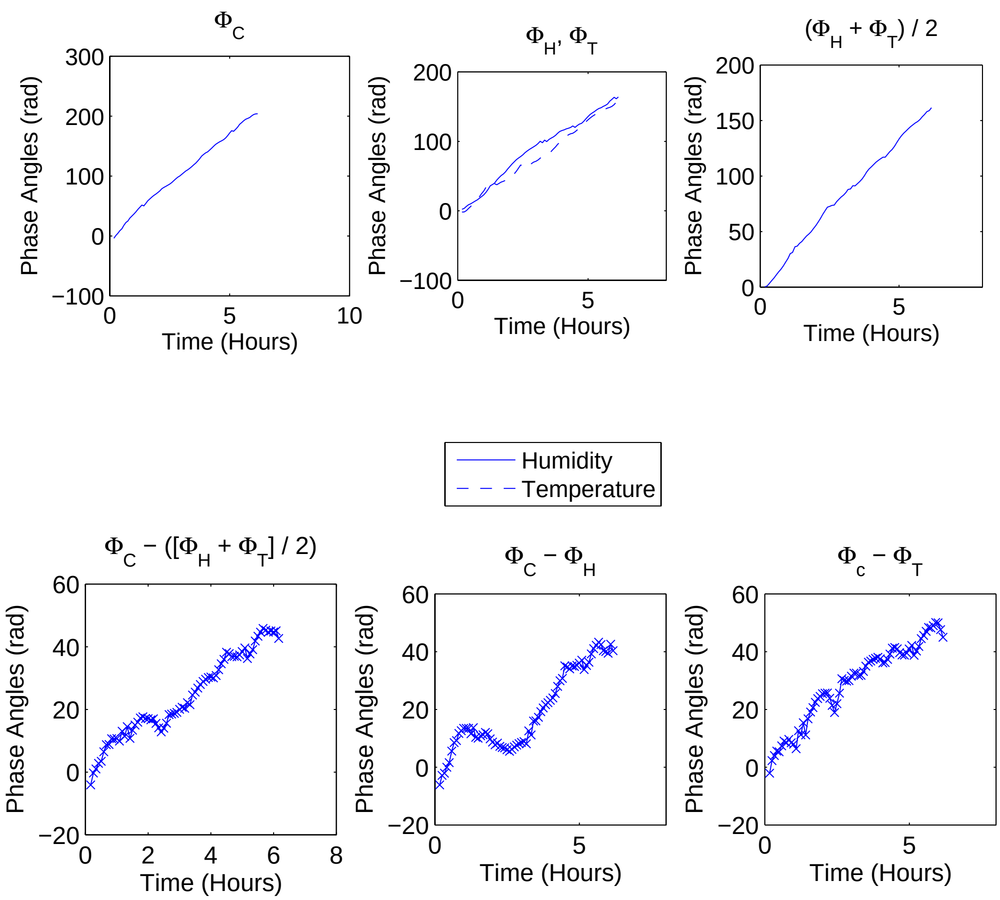

Figure 9. April 3, 2004 morning. Temperature dominates.

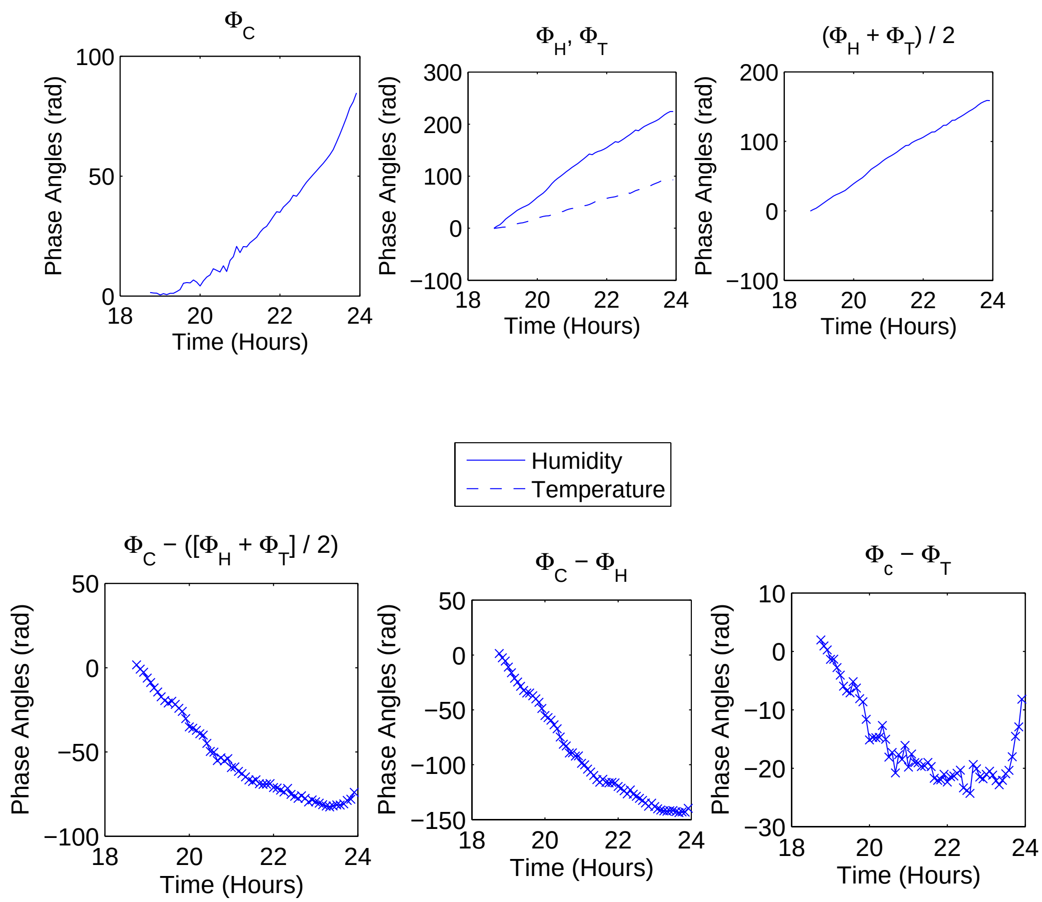

Figure 10. April 3, 2004 night. Humidity dominates.